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Abstract

Industry productivity is a weighted average of firm-level productivity. This productivity average can change due to i) changes in the distribution of firm-level productivity, ii) market share reallocation, iii) contributions of entering firms and iv) contributions of exiting firms. Several decomposition methods have been developed in order to appropriately identify the contributions of these four channels to industry productivity changes. This paper shows that widely-used decompositions yield biased contributions of surviving, entering and exiting firms. We also find biased contributions of within-firm productivity improvements and market share reallocation for surviving firms. Building on the cross-sectional decomposition proposed by Olley and Pakes (1996), we propose a decomposition that eliminates these biases. Our decomposition expresses its components in terms of moments of joint size and productivity distributions for three sets of firms: survivors, entrants and exitors. Using panel data on Slovenian manufacturing firms, we compare the results of different decompositions and find that new decomposition gives more realistic estimates of contributions of different channels to the change in the aggregate productivity.

Keywords: decomposition, aggregate productivity, distributional moments

JEL classifications: C10, O47

1 Introduction

The industry productivity is a weighted average of firm-level productivity. This productivity average can change due to i) changes in the distribution of firm-level productivity of surviving firms, ii) market share reallocation between surviving firms, iii) contribution of entering firms and iv) contribution of exiting firms. The first method that attempts to identify these four channels was developed by Bailey, Hulten and Campbell (1992). They distinguish between different contributions of surviving firms by fixing market share/productivity of firms, and attribute to entering and exiting firms their entire aggregate productivity weighted by overall market shares. Their method was later criticized as it attributes positive contribution to entering firms and negative contribution to exiting firms, irrespective of the difference between productivity of these firms and productivity of surviving firms. In order to eliminate these obvious biases in identification of contributions of entrants and exitors, Griliches and Regev (1995) and Foster, Haltiwanger and Krizan (2001) proposed improved varieties of Bailey *et al.* decomposition. Both of these methods introduced reference productivity values in the calculation of contributions of entering and exiting firms, and thus significantly reduced the biases in all four channels of aggregate productivity change. However, despite significant improvement in measurement of contributions of different groups of firms, the two decompositions did not eliminate them completely.

In this paper we argue that Foster et al. and Griliches and Regev decompositions do not identify correctly the contributions of different channels. We relate these biases to two key features of these methods. Both use ill-suited reference productivity values in the evaluation of contributions of entering and exiting firms, and fix weights (market shares) in division of contribution of surviving firms between productivity improvements and reallocation. The former distorts the distribution of change in the aggregate productivity between surviving, entering and exiting firms, whereas the latter distorts the two channels measuring contributions of within-firm productivity improvements and market share reallocation between surviving firms. We show that the two methods mix contributions of different groups of firms/channels, instead of reflecting only the contribution of channel in question. As a consequence, the size and the direction of biases for contributions of different channels depends on actual productivity and market share dynamics.

Due to these problems, we propose an alternative decomposition with components that do not mix contributions of different channels to aggregate productivity change. The proposed method relies on two features of definition of aggregate productivity: i) the aggregate productivity of any group of firms can be expressed as a weighted average of aggregate productivities of subgroups of firms and (ii) the well-known Olley-Pakes decomposition (1996), which decomposes the aggregate productivity into average productivity and covariance between market shares and productivity. The first feature allows us to express the component measuring the contribution of surviving firms as the change in aggregate productivity of surviving firms and the components measuring the contributions of entering and surviving firms as differences between aggregate productivity of these two sets of firms and corresponding aggregate productivity of surviving firms. Relying on the Olley-Pakes decomposition allows us to avoid time-invariant weights in identification of contribution of productivity improvements within firms and market share reallocation.

We illustrate differences between our and the Foster *et al.* and Griliches and Regev decomposition using a panel of Slovenian manufacturing firms that were active in the period 1995-2000. As shown by Bartelsman, Haltiwanger and Scarpetta (2007) and Polanec (2004), Slovenia is a convenient country for such comparison as the change in the aggregate productivity can be attributed to all four channels. In line with theoretical predictions regarding biases of existing decompositions, which suggest that fast growing sectors/industries, with increasing aggregate productivity of surviving firms, productivity advantage of surviving firms over entering and exiting firms, and higher overall market share of entrants, we find that existing methods yield a significantly higher contribution for entering firms and lower contribution for surviving firms than our method. The Griliches and Regev method yields lower bias for entering firms and higher for exiting firms than the Foster *et al.* method. We also find that these biases increase with length of time span and are generally robust to choice of productivity measures and weights. Finally, we extend the proposed decomposition to relate industry-level components to sector-level components and confirm previous results that intra-industry reallocation contributes more to aggregate productivity growth than inter-industry reallocation.

The remainder of the paper is organized in the following way. The next section reviews existing decompositions. In the third section we outline the new decomposition and discuss potential measurement biases. In the fourth section, we illustrate the difference between this method and alternative methods using real firm-level data and in the fifth section, we consider the relationships between the components of new decomposition at different levels of aggregation. In the last section we conclude.

2 Review of existing decompositions

The methods that decompose the change in aggregate productivity indices into components that reflect contributions of different processes/groups of firms are relatively recent in empirical industrial organization literature, coinciding with development of theories of economic growth with creative destruction that emphasize the importance of market shares reallocation for growth in the aggregate productivity. Following the seminal contribution of Bailey *et al.* (1992), the widely-used methods (Griliches and Regev, 1995; Foster *et al.*, 2001; Olley and Pakes, 1996) decompose the change of the standard index of aggregate productivity[1]:

$$\Phi_t = \sum_{i \in \Omega_t} s_{it}\varphi_{it}, \tag{1}$$

where s_{it} and φ_{it} denote the output (or employment) share of firm i in an industry in period t and its index of productivity, respectively, and Ω_t denotes the set of active firms in this period. In the remainder of this section, these methods are discussed in detail.

Bailey *et al.* (1992) propose to decompose the change in the aggregate productivity index between two periods in the following four components:

$$\Phi_2 - \Phi_1 = \sum_{i \in S} s_{i1}(\varphi_{i2} - \varphi_{i1}) + \sum_{i \in S}(s_{i2} - s_{i1})\varphi_{i2} + \sum_{i \in E} s_{i2}\varphi_{i2} - \sum_{i \in X} s_{i1}\varphi_{i1}, \tag{2}$$

where S, E and X denote the sets of surviving, entering and exiting firms, respectively. The first term on the right-hand side of equation (2) is called "the within component" as it aims to capture the contribution of productivity improvements within surviving firms. These improvements are weighted by market shares from initial period. The second term, the "between component," measures the contribution of shifts of market shares between surviving firms weighted by end period productivity indices. The last two terms measure the contributions of entering and exiting firms, calculated as sums of cross products between firms' market shares and productivities. The way these contributions are calculated is the main objection against this decomposition as it yields positive and negative contributions for entering and exiting firms, respectively, as long as there are some firms that decide to enter and exit.[2]

Griliches and Regev (1995) and Foster *et al.* (2001) propose alternative decompositions that introduce reference productivity values in calculation of contributions of entering and exiting firms. The Griliches and Regev decomposition is given by:

[1] Motivated by discrepancies between the standard index of aggregate productivity and the measure of welfare, Petrin and Levinsohn (2005) decompose the change of the Tornquist-Divisia index.

[2] These entry and exit terms can be expressed as $s_{E2}\Phi_{E2}$ and $s_{X1}\Phi_{X1}$.

$$\Phi_2 - \Phi_1 = \sum_{i \in S} \bar{s}_i(\varphi_{i2} - \varphi_{i1}) + \sum_{i \in S}(s_{i2} - s_{i1})(\bar{\varphi}_i - \bar{\Phi}) + + \sum_{i \in E} s_{i2}(\varphi_{i2} - \bar{\Phi}) - \sum_{i \in X} s_{i1}(\varphi_{i1} - \bar{\Phi}), \quad (3)$$

where $\bar{s}_i = (s_{i1}+s_{i2})/2$, $\bar{\varphi}_i = (\varphi_{i1}+\varphi_{i2})/2$ and $\bar{\Phi} = (\Phi_1+\Phi_2)/2$. The terms of this decomposition have the same economic interpretation as those of Bailey *et al.*, although the expressions of components are quite different. The productivity improvements of surviving firms are now weighted by time averages of market shares instead of initial period market shares, whereas the shifts of market shares are weighted by the difference between time averages of individual and aggregate productivity indices. These changes in terms that measure contributions for surviving firms are also reflected in terms that measure the contributions of entering and exiting firms. In contrast to the Bailey *et al.* decomposition, the contribution of entrants is now positive only if the aggregate productivity index of entrants in period 2 exceeds the time average of aggregate productivity index for all active firms and the contribution of exitors is negative only if the aggregate productivity of exitors in period 1 exceeds the time average of aggregate productivity index for all active firms.[3] These modifications reduce the biases inherent in measurement of contributions of entrants and exitors, although, as we show below, not completely.

Foster *et al.* suggest an alternative way to decompose the change of aggregate productivity index and introduce the reference productivity level in terms that capture the contributions of entry and exit:

$$\Phi_2 - \Phi_1 = \sum_{i \in S} s_{i1}(\varphi_{i2} - \varphi_{i1}) + \sum_{i \in S}(s_{i2} - s_{i1})(\varphi_{i1} - \Phi_1) + + \sum_{i \in S}(s_{i2} - s_{i1})(\varphi_{i2} - \varphi_{i1}) + \sum_{i \in E} s_{i2}(\varphi_{i2} - \Phi_1) - - \sum_{i \in X} s_{i1}(\varphi_{i1} - \Phi_1). \quad (4)$$

Their decomposition preserves the first term of Bailey *et al.* decomposition and modifies the remaining components. The process of reallocation of market shares between surviving firms is captured in two components. The first is also called the "between component" and measures the contribution of shifts in market shares weighted by the difference between initial individual and initial aggregate productivity index. This component is positive only when market shares move

[3] The contributions of entrants and exitors can be rewritten as $s_{E2}(\Phi_{E2} - \bar{\Phi})$ and -$s_{X1}(\Phi_{X1} - \bar{\Phi})$, where s_{E2} and s_{X1} denote the aggregate market shares of entrants and exitors and Φ_{E2} and Φ_{X1} denote the aggregate productivity indices of entrants and exitors.

in line with deviation of initial individual productivity index from initial aggregate productivity index. The second is the "cross or covariance component" and measures the covariance between changes of individual market shares and changes of individual productivity indices. This term is positive if changes in market shares and productivity indices move in the same direction and vice versa. The last two terms measure the contributions of entering and exiting firms. In contrast to the Griliches and Regev decomposition, the Foster *et al.* decomposition yields positive contribution for entrants if the aggregate productivity index for entrants exceeds the period 1 aggregate productivity index of all active firms (including exitors), and negative contribution for exitors if the aggregate productivity index for exitors exceeds the period 1 aggregate productivity index calculated for all firms. While both methods reduce the measurement biases, neither of the two eliminates them completely, which is a motivation for our paper.

The last decomposition that is commonly used in empirical literature was suggested by Olley and Pakes (1996). Unlike the methods described above, their decomposition distinguishes between the contributions of productivity improvements and reallocation using the moments of the joint distribution of firms' productivity indices and market shares. The static Olley-Pakes decomposition splits the aggregate productivity index in two components:

$$\begin{aligned} \Phi_t &= \bar{\varphi}_t + \sum_i (s_{it} - \bar{s}_t)(\varphi_{it} - \bar{\varphi}_t) \\ &= \bar{\varphi}_t + \operatorname{cov}(s_{it}, \varphi_{it}), \end{aligned} \tag{5}$$

where $\bar{\varphi}_t = \frac{1}{n_t}\sum_{i=1}^{n_t}\varphi_{it}$ is the unweighted firm productivity mean and $\bar{s}_t = \frac{1}{n_t}\sum_{i=1}^{n_t} s_{it}$ is the mean market share. Note that there is a slight abuse of notation with the definition of the cov operator, which would typically be multiplied by $1/n_t$. However, since s_{it}s are market shares, they essentially already incorporate the division by the number of firms n_t. Ignoring entry and exit of firms, the first difference of aggregate productivity index is:

$$\begin{aligned} \Delta\Phi = \Phi_2 - \Phi_1 &= (\bar{\varphi}_2 - \bar{\varphi}_1) + (\operatorname{cov}_2 - \operatorname{cov}_1) \\ &= \Delta\bar{\varphi} + \Delta\operatorname{cov}, \end{aligned}$$

where $\Delta\bar{\varphi}$ is the change in average productivity and represents the contribution of within-firm productivity improvements, while $\Delta\operatorname{cov}$ represents the contribution of market-share reallocation. Needless to say, this decomposition only distinguishes between the contributions of productivity improvements and reallocation, and thus does not allow us to distinguish between contributions of surviving, entering and exiting firms. However, as we show in the next section, the Olley-Pakes static decomposition of the change in aggregate productivity index can be extended to capture the contributions of these three sets of firms.

3 Dynamic Olley-Pakes decomposition with entry and exit

The decomposition that we propose in this section combines two ways of expressing the aggregate productivity. On one hand, the aggregate productivity index for a group of firms is a weighted mean of aggregate productivities of subgroups of firms:

$$\Phi_t = \sum_{g \in G} s_{gt} \Phi_{gt}, \ \sum_{g \in G} s_{gt} = 1$$

where s_{gt} and Φ_{gt} represent the aggregate market share and aggregate productivity of group g in period t. In particular, we shall express aggregate productivities in two subsequent periods as:

$$\Phi_1 = s_{S1}\Phi_{S1} + s_{X1}\Phi_{X1}, \tag{6}$$

$$\Phi_2 = s_{S2}\Phi_{S2} + s_{E2}\Phi_{E2}, \tag{7}$$

preserving the notation of different groups of firms.

Similarly, the static Olley-Pakes decomposition can be applied to any group of firms:

$$\Phi_{gt} = \bar{\varphi}_{gt} + \mathrm{cov}_{gt}, \tag{8}$$

where $\bar{\varphi}_{gt}$ is the unweighted average productivity for group g in period t and cov_{gt} is the covariance between market shares and productivity for this group of firms.

Using (6),(7) and (8), we rewrite the change in aggregate productivity index as:

$$\Delta\Phi = (\Phi_{S2} - \Phi_{S1}) + s_{E2}(\Phi_{E2} - \Phi_{S2}) + s_{X1}(\Phi_{S1} - \Phi_{X1}) \tag{9}$$

$$= \Delta\bar{\varphi}_S + \Delta\,\mathrm{cov}_S + s_{E2}(\Phi_{E2} - \Phi_{S2}) + s_{X1}(\Phi_{S1} - \Phi_{X1}) \tag{10}$$

In the first equality (9), we exploit only the first feature of the definition of aggregate productivity, which allows us to split the change of aggregate productivity into contributions of surviving, entering and exiting firms. The contribution of surviving firms is positive if aggregate productivity of these firms increases over time. The contribution of entering firms is positive only if aggregate or wieghted productivity of entering firms exceeds aggregate productivity of surviving firms in period 2, whereas the contribution of exiting firms is positive only if the aggregate productivity of surviving firms exceeds that of exiting firms in period 1. Both of these contributions are weighted by corresponding overall market shares. The second equality (10) exploits the static Olley-Pakes decomposition and splits the change in the weighted productivity for surviving firms into two components: $\Delta\bar{\varphi}_S$ and $\Delta\,\mathrm{cov}_S$ represent the contributions of productivity improvements and market share reallocations for the subgroup of surviving firms.[4]

Comparing (3), (4) and (10) reveals significant differences between the components of the proposed decomposition and the components of the Foster *et al.,* and Griliches and Regev decompositions. This is a consequence of two methodological differences. First, our decomposition yields positive contribution for entering and exiting firms only when aggregate productivity of these firms exceeds that of surviving firms in corresponding periods, while the other two decompositions compare aggregate productivity of entering and exiting firms to either aggregate productivity of all firms in initial period (Foster *et al.*) or the unweighted time average of aggregate productivity of all firms (Griliches and Regev). Second, we do not distinguish between contribution of change in productivity distribution from reallocation for survivors by fixing weights as the other two methods and follow instead the approach of Olley-Pakes decomposition and define reallocation only when covariance between market share and productivity increases. In the remainder of this section, we show that these methodological differences have important implications for the measurement of all four channels of aggregate productivity change and argue that, in general, the Foster *et al.* and Griliches and Regev decompositions yield biased contributions.

In Table 1 we show implications of choice of different reference productivity values (in entry and exit terms) by comparing the formulae for overall contributions of surviving, entering and exiting firms between the three decompositions. Ignoring further division of contribution of surviving firms allows us to express the components in terms of aggregate market shares and productivities. Our decomposition calculates the contribution of survivors as a change in the aggregate productivity, whereas the other two decompositions give weight to this change that is less than one when entrants (or exitors for Griliches and Regev) have positive overall market share, and contain terms that compare aggregate productivity of survivors and exiting firms

[4] The contributions of entrants and exitors can be split in two parts as well. The first part is positive when their unweighted average productivity is higher than that of the surviving firms they compete with (the term $s_{E2}\,(\bar{\varphi}_{E2} - \bar{\varphi}_{S2})$); second term is positive when the covariance between market share and productivity is higher for entrants than for the group of surviving firms (the term $s_{E2}(\mathrm{cov}_{E2} - \mathrm{cov}_{S2})$). This is clear in the case where both entrants and the surviving firms have the same unweighted average productivity. If more productive entrants have relatively higher market shares than more productive surviving firms – then this positively contributes to an increase in aggregate productivity. Exiting firms also contribute to aggregate productivity changes in the same way as entrants: a direct productivity effect, and an indirect market share reallocation effect between the exiting firms and surviving firms from period 1.

(or entering firms for Griliches and Regev). Since all methods decompose the same aggregate measure of productivity change, differences between components for survivors are reflected in components for entrants and exitors. While our decomposition attributes positive contribution to entry when aggregate productivity of entrants exceeds that of surviving firms in the end period (weighted by the overall market share of entrants), the other two decompositions also contain terms that reflect improvement in the aggregate productivity of surviving firms and terms that reflect productivity advantage of survivors over exitors in initial period. Similarly, our decomposition attributes positive contribution to exit when the aggregate productivity of survivors in the initial period exceeds the aggregate productivity of exitors (weighted by the aggregate market share of exitors), the Griliches and Regev decomposition attributes to exit also a part of the change in the aggregate productivity of survivors and aggregate productivity advantage of entrants over survivors. This comparison implies that Foster *et al.* and Griliches and Regev decompositions yield biased contributions for all three sets of firms. However, the size and the direction of these biases depend on features of firm dynamics of the actual data at hand. The contributions estimated by the existing decompositions may be either underestimated or overestimated, depending on the relations between aggregate productivities of different groups of firms and market shares of entrants and exitors. Nevertheless, in a highly relevant case of a fast growing industry all the terms that determine the biases work in the same direction.

Table 1: Comparison of Decompositions: Survivors vs. Entrants vs. Exitors

Group	Foster *et al.*	Griliches and Regev	DOPD
Survivors	$(1-s_{E2})(\Phi_{S2}-\Phi_{S1})$ $-2s_{X1}\bar{s}_{EX}(\Phi_{S1}-\Phi_{X1})$	$(1-\bar{s}_{EX})(\Phi_{S2}-\Phi_{S1})$ $-s_{X1}\frac{\Delta s_{EX}}{2}(\Phi_{S1}-\Phi_{X1})$ $+s_{E2}\frac{\Delta s_{EX}}{2}(\Phi_{E2}-\Phi_{S2})$	$\Phi_{S2}-\Phi_{S1}$
Entrants	$s_{E2}(\Phi_{E2}-\Phi_{S2})$ $+s_{E2}(\Phi_{S2}-\Phi_{S1})$ $+s_{E2}s_{X1}(\Phi_{S1}-\Phi_{X1})$	$s_{E2}(1-\frac{s_{E2}}{2})(\Phi_{E2}-\Phi_{S2})$ $+\frac{s_{E2}}{2}(\Phi_{S2}-\Phi_{S1})$ $+s_{E2}\frac{s_{X1}}{2}(\Phi_{S1}-\Phi_{X1})$	$s_{E2}(\Phi_{E2}-\Phi_{S2})$
Exitors	$s_{X1}(1-s_{X1})(\Phi_{S1}-\Phi_{X1})$	$s_{X1}(1-\frac{s_{X1}}{2})(\Phi_{S1}-\Phi_{X1})$ $+\frac{s_{X1}}{2}(\Phi_{S2}-\Phi_{S1})$ $+s_{X1}\frac{s_{E2}}{2}(\Phi_{E2}-\Phi_{S2})$	$s_{X1}(\Phi_{S1}-\Phi_{X1})$

Notes: $\bar{s}_{EX}=\frac{1}{2}(s_{X1}+s_{E2})$, $\Delta s_{EX}=s_{E2}-s_{X1}$.

In particular, consider an industry with growing aggregate productivity of surviving firms ($\Phi_{S2}>\Phi_{S1}$), positive aggregate productivity advantage of surviving firms over entering and exiting firms in corresponding periods ($\Phi_{S2}>\Phi_{E2}$, $\Phi_{S1}>\Phi_{X1}$), and market share of entrants that exceeds market share of exiting firms ($s_{E2}>s_{X1}>0$). These assumptions, consistent with firm dynamics during transition, imply that both existing decompositions yield downward-biased contribution for surviving firms and upward biased contribution for entering firms. Further, the Foster *et al.* decomposition yields downard-biased contribution for exiting firms, whereas the opposite holds for the Griliches and Regev decomposition. Clearly, the absolute values of these biases increase with the speed of productivity improvements of surviving firms, the productivity advantage of surviving firms over entering and exiting firms and the difference between market shares of entering and exiting firms. This has important implication for the relationship between the size of these biases and elapsed time between initial and end period of decomposition since over longer time periods both productivity improvements of surviving firms and market shares of entering and exiting firms increase.[5]

The biases introduced by ill-suited choice of reference productivity values to measurement of contribution of surviving firms carry over to components that reflect productivity growth and market share reallocation between surviving firms. However, these biases are further exacerbated by choice of fixed weights in identification of these components in the Foster et al. and Griliches and Regev decompositions. This is shown in Table 2, where we compare the components of different decompositions for these two channels. Note, first, that the change in productivity distribution of surviving firms is fully captured only by our decomposition. The other two decompositions weight these improvements either by the overall market share of surviving firms in initial period (Foster *et al.*) or by the time average of market share of surviving firms (Griliches and Regev). Moreover, the components of these two decompositions partly reflect the changes in covariance between market share and productivity of surviving firms. For example, the component of Foster *et al.* decomposition contains a term that compares covariance between period 1 market share and period 2 productivity ($\widehat{cov}_{S12}$) with period 1 covariance between market share and productivity. Next, only our decomposition defines market share reallocation between surviving firms with unweighted change in covariance between market share and productivity. The component of Foster *et al.* decomposition contains only a part of this term, and subtracts two additional terms that reflect changes in average productivity of surviving firms and difference between aggregate productivities of surviving and exiting firms. Similarly, the component of Griliches and Regev decomposition contains four terms, two of which reflect changes in covariance over time and two reflect differences between aggregate productivities of surviving, entering and exiting firms. Mixing the terms that capture contributions of different channels suggests that the existing decompositions yield biased contributions also for components that aim to capture productivity improvements and reallocation for the group of

[5]The productivity advantage of survivors over entrants and exitors does not necessarily increase with elapsed time.

surviving firms. The size and direction of these biases depends on the features of firm dynamics. Without a large set of assumptions regarding the relations between different components, it is not possible to establish the direction of bias. Even for the example of growing industry with additional assumption on ranking of covariances ($cov_{S2} > \widehat{cov}_{S21} > \widehat{cov}_{S12} > cov_{S1}$), we can be certain only of downward bias for the components that measure reallocation, but not for the components that capture changes in productivity distribution. Hence, we illustrate these biases using a real firm-level data.

Table 2: Comparison of Decompositions: Productivity Growth vs. Reallocation

Process	Foster *et al.*	Griliches and Regev	DOPD
Productivity growth	$(1 - s_{X1})(\bar{\varphi}_{S2} - \bar{\varphi}_{S1})$ $+(1 - s_{X1})(\widehat{cov}_{S12} - cov_{S1})$	$(1 - \bar{s}_{EX})(\bar{\varphi}_{S2} - \bar{\varphi}_{S1})$ $+\frac{1}{2}(1 - s_{E2})(cov_{S2} - \widehat{cov}_{S21})$ $+\frac{1}{2}(1 - s_{X1})(\widehat{cov}_{S12} - cov_{S1})$	$\bar{\varphi}_{S2} - \bar{\varphi}_{S1}$
Reallocation	$(1 - s_{E2})cov_{S2} - (1 - s_{X1})\widehat{cov}_{S12}$ $-\Delta s_{EX}(\bar{\varphi}_{S2} - \bar{\varphi}_{S1})$ $-\Delta s_{EX}s_{X1}(\Phi_{S1} - \Phi_{X1})$	$\frac{1}{2}(cov_{S2} - cov_{S1})$ $+\frac{1}{2}(1 - s_{E2})\widehat{cov}_{S21} - \frac{1}{2}(1 - s_{X1})\widehat{cov}_{S12}$ $-s_{X1}\frac{\Delta s_{EX}}{2}(\Phi_{S1} - \Phi_{X1})$ $-s_{E2}\frac{\Delta s_{EX}}{2}(\Phi_{S2} - \Phi_{E2})$	$cov_{S2} - cov_{S1}$

Notes: $\widehat{cov}_{S12} = cov(s_{S1}, \varphi_{S2})$, $\widehat{cov}_{S21} = cov(s_{S2}, \varphi_{S1})$, $\bar{s}_{EX} = \frac{1}{2}(s_{E2} + s_{X1})$, $\Delta s_{EX} = s_{E2} - s_{X1}$.

4 Comparison of decompositions with real data

For comparison of decompositions, we use accounting data for a panel of Slovenian manufacturing firms (NACE 2-digit industries 15-37) that were active in the period 1995-2000.[6] During this period the manufacturing sector (and the rest of the economy) went through significant structural changes that were triggered by economic reforms adopted in the late 1980s and early 1990s (e.g. liberalization of prices and wages, deregulation of entry, and privatization of state-owned firms). The shock caused by economic reforms initially led to large decline in both aggregate output and labor productivity, followed by fast reversal to high growth, which persisted through entire period of analysis. Previous empirical research using Slovenian manufacturing data (Polanec, 2004; Bartelsman, Haltiwanger and Scarpetta 2007) established that an important part of aggregate productivity growth in the sector could be attributed to market share reallocation between surviving, entering and exiting firms. Evidence that all four channels contributed to the growth in aggregate productivity makes our data set ideal for illustration of differences between methods. Given these features of the data, we expect the direction of the

[6] We are greatful to the Slovenian Agency for Public Legal Records and Related Services (AJPES) for providing the data.

biases to coincide with those discussed in the example of a growing industry.

Description of data

The data set contains information on firm identity, year of reporting, annual sales, costs of material and services, nominal physical capital and employment. From these we calculate all standard measures of labor and total factor productivity. Prior to calculation of productivity measures, we deflate the firm revenues and material costs by NACE 2-digit producer price indices and physical capital by investment goods price index. The reported number of employees is calculated from the annual number of hours worked by all workers.

Table 3 summarizes the descriptive statistics for the dataset of active firms in Slovenian manufacturing. In order to include the same set of firms in all decompositions, we require that a firm employs at least one worker, engages positive physical capital and generates positive value added. Clearly, this definition of an active firm is stricter than the legal requirements. The number of firms that comply with our definition increased by 18.4 percent between 1995 and 2000, from 3,867 to 4,580. Among these firms were 2,677 survivors, 1,903 entrants and 1,191 exitors. At the same time, the average size of active firms, measured by number of employees, decreased from 60.1 to 45.2 employees, which is mainly a consequence of entry of smaller new firms and partly due to a reduction in size of surviving firms.[7] The downsizing of surviving firms and the exit of firms also contributed to a decline of aggregate employment by 11.2 percent, from 233 to 207 thousand, despite the fact that the real aggregate sales, real aggregate value added and real aggregate physical capital increased by 46.1 percent, 45.8 percent and 25.3 percent, respectively.

Choice of productivity indices and weights

Foster *et al.* (2001) show that the size of different components of aggregate productivity decompositions depends not only on the choice of the method of decomposition, but also on the choice of productivity measure and weights. For this reason we compare the results of different decompositions using two indices of firm-level productivity with corresponding weights.[8] The first measure of productivity is the standard labor productivity, defined as a ratio between real

[7] The average employment of surviving firms between 1995 and 2000 declined from 67.4 to 59.4 employees. In 2000, the average employment of entrants was 25.1 employees.

[8] The productivity indices are given in logarithms in order to avoid measurement bias when calculating the contributions of surviving firms for our decomposition. If productivity index is not given in logarithms, the same percentage change in productivity of all surviving firms would be split into equal contributions of the average productivity improvements and the change in covariance of surviving firms. Since all firms improve productivity to the same extent, the contribution of reallocation should be equal to zero, which is achieved when productivity index is given in logarithms.

Table 3: Descriptive statistics for Slovenian manufacturing firms in 1995 and 2000

	Year	
Set of firms	1995	2000
Number of all firms	3867	4580
Number of survivors	2677	2677
Number of entrants		1903
Number of exitors	1191	
Variable		
Average employment	60.1	45.2
Aggregate employment [in thousand]	232	206
Real aggregate value added [in bln. SIT]	425	620
Real aggregate output [in bln. SIT]	1520	2220
Real aggregate physical capital [in bln. SIT]	862	1080

Notes: The real value added and output are deflated by corresponding. NACE 2-digit industry producer price index. and the real physical capital is deflated by investment goods price index
Source: SORS and own calculations.

value added and employment, and the second measure, using the definition of Foster, Haltiwanger and Syverson (2008), is the revenue total factor productivity, which is obtained as a regression residual:

$$\ln TFP_{it} = \ln Y_{it} - \hat{\alpha} \ln K_{it} - \hat{\beta} \ln L_{it}, \tag{11}$$

where Y_{it}, K_{it} and L_{it} denote the real value added, real capital and employment of firm i in period t, respectively, and $\hat{\alpha}$ and $\hat{\beta}$ denote the estimates of regression coefficients for capital and labor.[9] In order to obtain consistent estimates of log of TFP for the sample of all active firms in the manufacturing sector, the regression equation includes both 2-digit NACE industry and annual time dummies.

Clearly, measurement of labor and total factor productivity affects the results of different decompositions. One limitation of our estimates of productivity indices is use of 2-digit NACE industry producer price indices in calculations of real value added. The evidence on presence of price dispersion within broad industries in the U.S. by Abbott (1992) and more recent evidence of price dispersion for producers of fairly homogenous products by Foster *et al.* (2008) suggests that our assumption may lead to mismeasurement of both labor and total factor productivity. In particular, Foster *et al.* (2008) find that entering firms charge lower prices than incumbent firms, while price differences between surviving and exiting firms are not statistically significant. This finding implies that all decompositions should give downward biased contributions for entering

[9] For example, Bartelsman, Haltiwanger and Scarpetta (2007) point out that classical measurement error in measures of productivity may decrease the covariance between market shares and log of productivity and thus reduce the contribution of reallocation to aggregate productivity growth.

firms. However, given their limited sample of products, it is not clear if their findings can be generalized to universe of manufacturing firms.[10] Moreover, since contribution of entering firms is downward biased in all decompositions, we believe that differences between decompositions are likely to be preserved.

Next issue in estimation of TFP is the choice of variables used in regression equation. Baily *et al.* (1992), Foster *et al.* (2001) and many others estimate the productivity indices using an alternative regression equation with a log of sales as the dependent variable and log of physical capital, employment and material costs as the explanatory variables. This approach is often favored because it does not impose unitary elasticity between sales and material costs. However, Petrin and Levinsohn (2005) show that this measure of productivity is greatly at odds with their measure of welfare (Tornquist-Divisia index) and suggest to obtain the residuals from regression equation given in (11).[11]

Finally, there are different methods for estimation of elasticities of production function. The ordinary least squares method is known to yield biased estimates of coefficients due to higher average productivity and average size of surviving firms (selection bias) and due to the fact that firms with higher productivity engage greater quantities of production factors (simultaneity bias). In order to reduce these measurement biases, several methods were suggested in the literature, ranging from instrumental variables techniques to structural estimation of production functions. Olley and Pakes (1996) proposed a semiparametric estimation method that aims to correct for both selection and simultaneity bias. They use a simple test that gives an indication of the magnitude of selection bias. They compare the OLS coefficients for full and balanced samples of U.S. firms producing telecommunications equipment and find that the coefficient for physical capital is significantly lower for the balanced sample.[12] This result leads them to a conclusion that the selection bias affects the estimates of productivity index. We make a similar comparison for the set of Slovenian manufacturing firms and find statistically insignificant effect of selection on the estimated regression coefficients.[13] A possible reason for lower selection bias may be related to the fact that the key characteristics of firms do not differ significantly between the two samples of firms.[14] Insignificant effect of selection bias on the estimates of regression coefficients

[10] Foster *et al.* (2008) analyze prices of firms that produce 11 fairly homogenous products.

[11] We have also estimated total factor productivity as a residual in a regression of log of sales on log of physical capital, employment and material costs. While the estimates of TFP are lower in absolute values, all qualitative findings regarding differences in the size of components of different decompositions are preserved.

[12] Olley and Pakes (1996) estimated the OLS capital coefficient with value added as the dependent variable 0.308 in a full sample (with entering and exiting firms) and 0.163 in a balanced sample (only those firms that are active over the entire period).

[13] The coefficient for physical capital estimated for the full sample (0.218) is lower than the same coefficient estimated for the balanced sample of firms (0.221), whereas the coefficient for employment for the full sample (0.796) is greater than the coefficient for the balanced sample (0.768).

[14] The full and the balanced sample consist of 19166 and 14826 observations. The average employment in all

is also reflected in the estimated coefficients of production function using the semiparametric procedures proposed by Levinsohn and Petrin (2003), which yield decreasing returns to scale and lower coefficient for physical capital than the OLS. Moreover, Van Biesebroeck (2004) finds high correlations between different across different estimates of TFP and suggests that these differences are unlikely to be first order. For this reason we use the estimates of firms' productivity indices as residuals from the OLS regressions. Note, however, that qualitative differences between decompositions are likely to be preserved.

A related issue is the choice of weights. In the empirical literature it is common to use the weights that correspond to the measure of productivity. We follow this approach and use the employment shares as weights for the log of labor productivity. For the log of TFP we follow Petrin and Levinsohn (2005) and use value added shares as weights. In this way, we look at correspondence between both capital and labor on one hand and TFP on the other hand.[15]

Results

Let us now turn to comparison of the results of our decomposition to the results of Foster *et al.* and Griliches and Regev decompositions. In the top panels of Tables 4, 5 and 6 we present the results of decompositions using the log of labor productivity, whereas the bottom panels show the decompositions for the log of revenue total factor productivity. The middle columns give the components of decompositions and the last column (denoted total) reports the change of aggregate productivity index. The decompositions are made between 1995 and all subsequent years until 2000 in order to illustrate the variation of measurement biases with the length of time span.

Focusing on the overall contributions of surviving, entering and exiting firms, we find that Foster *et al.* and Griliches and Regev decompositions consistently underestimate the contribution of surviving firms and overestimate the contribution of entering firms. We also that the Foster *et al.* decomposition modestly underestimates the contribution of exiting firms, whereas the Griliches and Regev decomposition overestimates their contribution. For example, using the log of labor productivity as a productivity index and comparing the results over the period 1995-2000, we find that aggregate productivity increased by 0.5033, out of which Foster *et al.* decomposition attributes 0.3447 surviving firms, 0.0990 to entrants and 0.0596

years is 58.97 for the full sample and 65.21 for the balanced sample of firms. The average of log of value added per employee for the full sample is 7.58 and 7.69 for the balanced sample of firms.

[15] Note also that our data show relatively small differences between the change of our measure of TFP and the Tornquist-Divisia index. The difference in definitions between these two indices is that the Tornquist-Divisia index uses corrections for indirect effects of productivity improvements on other firms in industry, whereas our measure does not. Since we find small differences between the two productivity indices, we believe that corrections for indirect effects emphasized by Petrin and Levinsohn (2005) are relatively small.

to exiting firms. The Griliches and Regev decomposition attributes a similar contribution to surviving firms, 0.3465, although the respective contributions of entrants and exitors, 0.0408 and 0.1160, are quite different due to use of a different reference aggregate productivity index. Our decomposition yields significantly higher overall contribution of surviving firms, 0.4490, a negative contribution of entering firms, -0.0225, and an intermediate contribution of exiting firms, 0.0768. These results confirm theoretical prediction about the direction of measurement biases for the Foster *et al.* and Griliches and Regev decompositions in a growing economy. A further comparison of results reveals that the absolute size of measurement biases monotonically increases with length of time span. For example, for the two-year period (1995-1997), the measurement biases for the contributions of surviving, entering and exiting firms for the Foster *et al.* decomposition are -0.033, 0.0403 and -0.0072, respectively, whereas over the five-year period (1995-2000), the corresponding biases are -0.1047, 0.1215 and -0.0172. Over the two-year period, the biases for surviving, entering and exiting firms for the Griliches and Regev decomposition are -0.0353, 0.0218 and 0.0145 and for the five-year period these are -0.1025, 0.0663 and 0.0392, respectively. Finally, note that qualitative features of observed measurement biases carry over to alternative productivity indices and sets of weights[16], which we illustrate for the log of total productivity and shares of value added as weights in the bottom panels of Tables 4, 5 and 6.[17]

Next we compare the contributions that the three decompositions attribute to the changes in productivity distribution and market share reallocation of surviving firms. Looking at five-year decompositions for the log of labor productivity, we can see small differences between the components that capture within-firm productivity improvements, ranging between 0.3095 for the Griliches and Regev decomposition and 0.3286 for our decomposition. On the other hand, our decomposition attributes 0.1204 to market share reallocation, whereas the Foster *et al.* and Griliches and Regev decompositions attributes only 0.0285 and 0.0370, respectively. The smaller bias for within-firm productivity improvements is consistent with theoretical discussion, which showed that the latter two decompositions atribute part of change in covariance to productivty improvement and subtract from the component measuring reallocation terms that are related to average productivity improvement of surviving firms and aggregate productivity advantage of surviving firms over entering and exiting firms. As above, we find that these biases increase with length of time span. They are not, however, robust to the choice of productivity and the set of weights. For the log of TFP with value added shares as weights, we find that for the five-year

[16]We have performed robustness checks using alternative sets of weights (sales and employment shares) for two productivity indices: log of TFP obtained from a regression of log of sales on log of capital, labor and material costs and log of TFP obtained from a regression of log value added on log of capital and labor.

[17]Note that since we calculate the log of TFP as a residual from a regression of the log of value added on the log of capital and labor, the changes in the aggregate log of TFP and the contributions of different sets of firms are always smaller than the corresponding changes in the aggregate log of labor productivity.

Table 4: Foster, Haltiwanger and Krizan decomposition

Year	Within	Between	Cross	Entry	Exit	Total
Log of value added per employee						
1996	0.1093	0.0223	-0.0086	-0.0221	0.0311	0.1320
1997	0.2336	0.0324	-0.0139	0.0141	0.0446	0.3108
1998	0.2345	0.0384	-0.0170	0.0333	0.0595	0.3488
1999	0.2874	0.0390	-0.0134	0.0599	0.0575	0.4305
2000	0.3162	0.0418	-0.0133	0.0990	0.0596	0.5033
Log of total factor productivity						
Year	Within	Between	Cross	Entry	Exit	Total
1996	0.0521	-0.0247	0.0874	0.0057	0.0073	0.1279
1997	0.1480	-0.0229	0.0890	0.0292	0.0185	0.2618
1998	0.1362	-0.0190	0.0900	0.0265	0.0257	0.2593
1999	0.1724	-0.0140	0.1106	0.0662	0.0213	0.3565
2000	0.1911	-0.0225	0.1202	0.0895	0.0229	0.4013

Notes: The aggregate value added per employee is calculated with employment shares as weights. The aggregate TFP is obtained from regression of log of value added on the log of physical capital and labor using value added as weights. The reference period for calculation of the change of aggregate productivity index is 1995.
Source: AJPES and own calculations.

period, the downward bias in the measurement of the contribution of surviving firms for the Foster *et al.* decomposition results from underestimation of the contribution of within-firm productivity improvements (-0.0847), whereas for the Griliches and Regev decomposition the bias is mainly a result of underestimation of the contribution of reallocation (-0.0498). Nevertheless, it is important to note that these differences in contributions are caused by sensitivity of the Foster *et al.* decomposition to alternative measure of productivity rather than the proposed decomposition.

Note that our decomposition allows us to splits the contributions of entering and exiting firms into direct and indirect contributions. For this purpose, Table 6 reports both contribution of unweighted and weighted productivity differences. We find that the direct contribution of entrants is always negative, irrespective of the choice of productivity index and time span. Since we calculate this component as a weighted difference between the average productivity of entering and exiting firms, this result implies that entering firms exhibit lower average productivity than surviving firms. The indirect contribution of entrants is always positive for the log of total factor productivity and mixed for the log of labor productivity. Relatively small values of these components suggests there are small differences between covariances of entering and surviving

Table 5: Griliches and Regev decomposition

Log of value added per employee					
Year	Within	Between	Entry	Exit	Total
1996	0.1050	0.0164	-0.0249	0.0355	0.1320
1997	0.2266	0.0223	-0.0044	0.0663	0.3108
1998	0.2260	0.0252	0.0070	0.0906	0.3488
1999	0.2807	0.0341	0.0173	0.0983	0.4305
2000	0.3095	0.0370	0.0408	0.1160	0.5033
Log of total factor productivity					
Year	Within	Between	Entry	Exit	Total
1996	0.0959	0.0178	0.0039	0.0103	0.1279
1997	0.1925	0.0213	0.0159	0.0321	0.2618
1998	0.1812	0.0261	0.0094	0.0426	0.2593
1999	0.2276	0.0475	0.0338	0.0475	0.3565
2000	0.2512	0.0457	0.0456	0.0588	0.4013

Notes: The aggregate value added per employee is calculated with employment shares as weights. The aggregate TFP is obtained from regression of log of value added on the log of physical capital and labor using value added as weights. The reference period for calculation of the change of aggregate productivity index is 1995.
Source: AJPES and own calculations.

firms. The direct contribution of exitors is the largest single component for both productivity indices. A large positive value for this term implies that the exiting firms had lower productivity than the surviving firms and thereby contribute to higher aggregate productivity. Moreover, the direct contribution of exiting firms increases with length of time span. In theoretical analysis in Appendix we show that the contribution of exiting firms increases with time span because the share of exiting firms increases and because productivity of exiting firms is lower than that of surviving firms.[18] On the other hand, the indirect contribution of exiting firms is always negative, which implies greater covariance between size and productivity for exiting firms than for surviving firms.

5 Extension: Sector vs. industry-level decomposition

The proposed decomposition can be applied at any level of aggregation of firms. Moreover, the components of decomposition, say, at sectoral level can be expressed in terms of components at the level of industries. In this section, we show the relations between components of decomposition at the sectoral and industry level under the assumption that firms do not shift their focus

[18] The variation of size of components with length of time span is further discussed in Appendix.

Table 6: Dynamic Olley-Pakes decomposition with entry and exit

Log of valued added per employee

	Surviving firms		Entering firms		Exiting firms		
Year	Change in Unweighted Productivity	Change in Covariance	Unweighted Productivity Difference Ent. vs. Sur.	Weighted Productivity Difference Ent. vs. Sur.	Unweighted Productivity Difference Sur. vs. Exit.	Weighted Productivity Difference Sur. vs. Exit.	Total
1996	0.1094	0.0183	-0.0165	-0.0290	0.0412	0.0333	0.1320
1997	0.2219	0.0632	-0.0218	-0.0261	0.0714	0.0518	0.3108
1998	0.2508	0.0483	-0.0265	-0.0228	0.0849	0.0724	0.3488
1999	0.3174	0.0736	-0.0298	-0.0315	0.0822	0.0710	0.4305
2000	0.3286	0.1204	-0.0291	-0.0225	0.0908	0.0768	0.5033

Log of total factor productivity

	Surviving firms		Entering firms		Exiting firms		
Year	Change in Unweighted Productivity	Change in Covariance	Unweighted Productivity Difference Ent. vs. Surv.	Weighted Productivity Difference Ent. vs. Surv.	Unweighted Productivity Difference Surv. vs. Exit.	Weighted Productivity Difference Surv. vs. Exit.	Total
1996	0.1002	0.0178	-0.0078	0.0021	0.0255	0.0077	0.1279
1997	0.1958	0.0426	-0.0086	0.0028	0.0427	0.0206	0.2618
1998	0.2145	0.0242	-0.0106	-0.0089	0.0508	0.0295	0.2593
1999	0.2671	0.0626	-0.0071	0.0017	0.0507	0.0250	0.3565
2000	0.2758	0.0955	-0.0072	0.0021	0.0572	0.0279	0.4013

Notes: The aggregate value added per employee is calculated with employment shares as weights. The aggregate TFP is obtained from regression of log of value added on the log of physical capital and labor using value added as weights. The reference period for calculation of the change of aggregate productivity index is 1995.
Source: AJPES and own calculations.

of market activity and remain in the same industry over time. By assuming this we eliminate a channel of inter-industry reallocation.[19] However, even in this simplified case the components of decomposition at the sectoral level that capture reallocation can not be expressed as simple weighted averages of industry-level components, but rather as a sum of two sets of components reflecting intra and inter-industry market reallocations.

The components at the sectoral level can be expressed in the following way. The change in the average productivity of surviving firms in a sector can be expressed as a simple weighted average of industry-level change in average productivity:

$$\bar{\varphi}_{S2} - \bar{\varphi}_{S1} = \sum_{j=1}^{J} \bar{s}_{Sj}(\bar{\varphi}_{Sj2} - \bar{\varphi}_{Sj1}), \tag{12}$$

where $\bar{\varphi}_{Sjt}$ is the average productivity of surviving firms in period t in industry j and $\bar{s}_{Sj}$ is the weight given to industry j, calculated as a ratio between the number of surviving firms in

[19]The share of firms that change the focus of economic activity is relatively small and the bias due to this type of inter-industry reallocation is small as well.

industry j and the total number of surviving firms in the sector, n_{Sj}/n_S.

The second of sectoral decomposition captures market share reallocation between surviving firms and consists of two terms. The first part measures the direct contribution of the intra-industry market share and productivity shifts, whereas the second part captures the indirect contribution of the intra-industry shifts that are reflected in inter-industry shifts in market shares and productivities. Expressed in this manner, the change in covariance for surviving firms of the entire sector is the following sum:

$$cov_{S2} - cov_{S1} = \sum_{j=1}^{J} \bar{s}_{Sj}(cov_{Sj2} - cov_{Sj1}) + (cov_{S2}^{Ind} - cov_{S1}^{Ind}), \tag{13}$$

where cov_{Stj} denotes the intra-industry covariance for the set of surviving firms in industry j in period t and cov_{St}^{Ind} denotes the inter-industry covariance. Note that the intra-industry changes of covariances are again weighted by the share of the number of surviving firms in a given industry relative to the total number of surviving firms. The second term of the right-hand side of equation (13) is the change of inter-industry covariance between market shares and aggregate productivities. The inter-industry covariance is defined as the following sum:

$$cov_{St}^{Ind} = \sum\nolimits_{j=1}^{J} (s_{Sjt} - \bar{s}_{Sj})(\Phi_{Sjt} - \Phi_{St}), \tag{14}$$

where s_{Sjt} represents the market share of surviving firms in industry j in aggregate output of surviving firms in the sector in period t. This term is positive when more productive industries tend to have above average market shares. Hence, the change in inter-industry covariance for surviving firms is positive when industry market shares shift in line with shifts in industry-level aggregate productivities.[20]

The sectoral-level contribution of entrants to change in the aggregate productivity can be expressed as a sum of two components:

$$s_{E2}(\Phi_{E2} - \Phi_{S2}) = \sum_{j=1}^{J} s_{2j} s_{E2j}(\Phi_{Ej2} - \Phi_{Sj2}) + \sum_{j=1}^{J} s_{j2} s_{Ej2}((\Phi_{Sj2} - \Phi_{Ej2}) - (\Phi_{S2} - \Phi_{E2})). \tag{15}$$

Here s_{j2} denotes the market share of firms in industry j in aggregate output of all firms in the sector in period 2, s_{Ej2} is the market share of entrants in industry j in aggregate output of

[20]Since aggregate productivity can be expressed as a sum of average productivity and covariance between market share and productivity, the change in inter-industry covariance can be further split into the change of covariance between industry market shares and average productivities and the change in covariance between industry market shares and intra-industry covariances. These two components capture the indirect effects of shifts in industry-level average productivities and covariances on sectoral covariance.

firms in industry j. A product of these two terms is the market share of entrants in industry j in aggregate output of all firms in the sector. The first term on the right-hand side of equation (15) measures the intra-industry contribution of entrants, whereas the second term measures the inter-industry contribution of entrants. The former is calculated as a sum of industry-level contributions of entrants weighted by industry-level market shares, whereas the latter is calculated as a weighted sum of deviations of industry-level aggregate productivity premia of surviving and entering firms from the difference of productivity between surviving and entering firms in the sector weighted by the industry market shares of entrants in the sector.

The sectoral-level contribution of exiting firms to change in the aggregate productivity can be decomposed along the same lines:

$$s_{X1}(\Phi_{S1} - \Phi_{X1}) = \sum_{j=1}^{J} s_{j1} s_{Xj1}(\Phi_{Sj1} - \Phi_{Xj1}) + \sum_{j=1}^{J} s_{j1} s_{Xj1}((\Phi_{Xj1} - \Phi_{Sj1}) - (\Phi_{X1} - \Phi_{S1})), \quad (16)$$

where s_{j1} denotes the market share of all firms in industry j in aggregate output of all firms in the sector in period 1 and s_{Xj1} denotes the market share of exitors in industry j in aggregate output of all firms in industry j. The interpretations of the right-hand side terms of equation (16) is analogous to that for entering firms.

Let us now apply this sectoral-level decomposition on the data set of Slovenian manufacturing firms. For the sake of brevity, we only report results for this decomposition for the entire 1995-2000 period and one productivity index (log of total factor productivity).[21] The top panel of Table 7 shows the results of industry-level decomposition for all NACE 2-digit industries in Slovenian manufacturing sector. As before, the middle columns give the absolute values of components of the new decomposition and the last column reports industry-specific change in aggregate productivity. The results reveal substantial heterogeneity across industries in terms of aggregate productivity improvements and components that comprise these shifts. For example, some industries exhibit very little improvement in aggregate productivity (e.g. Food, Beverages and Tobacco; Apparel), while others exhibit large improvements (e.g. Basic metals, Paper). Since surviving firms contribute most to the change in aggregate productivity, lower overall aggregate productivity improvement typically coincides with lower aggregate productivity improvement for surviving firms. With the exception of some industries, the contributions of entering firms are typically small and the signs are just as often negative as positive. The contributions of exiting firms show more consistency. In all industries the contribution of exiting firms is positive, which implies that firms that exit systematically exhibit lower aggregate

[21] Note that since some firms shift focus of economic activity, we attribute to such firms the industry in which they operated predominantly. In our sample 2.66 percent of firms changed the main industry of operations.

productivity than surviving firms.

The bottom panel of Table 7 shows the components of the sectoral decomposition. The overall change in aggregate productivity, 0.4013, is split into contribution of intra-industry components, 0.3675, and inter-industry productivity components, 0.0338. This result suggests that intra-industry components are significantly larger than the inter-industry components. Among the intra-industry components dominates the direct contribution of surviving firms, 0.2758, followed by the indirect contribution of surviving firms, 0.0570, and the direct contribution of exiting firms, 0.0569. The inter-industry components are smaller in comparison to the intra-industry components. The only component that stands out is the indirect contribution of surviving firms, 0.0385, which measures the change in aggregate productivity due to improved covariance between industry-level market shares and aggregate productivities of surviving firms. These results are consistent with previous findings of key importance of intra-industry reallocation in growth of aggregate productivity (Foster et al., 2001).

Table 7: Sectoral dynamic Olley-Pakes decomposition with entry and exit

	Surviving firms		Entering firms	Exiting firms	
Industry	Change in unweighted Productivity	Change in Covariance	Weighted Productivity Difference Ent. vs. Sur.	Weighted Productivity Difference Sur. vs. Exit.	Total
Food Tobacco (15+16)	-0.0812	0.0888	-0.0337	0.0382	0.0122
Textilles (17)	0.2512	0.1322	0.0743	-0.0003	0.4574
Apparel (18)	0.2083	-0.1783	0.0388	-0.0515	0.0173
Leather (19)	0.2197	0.1428	0.0193	-0.0016	0.3802
Wood (20)	0.2655	-0.0716	0.0977	0.0514	0.3430
Paper (21)	0.4992	0.3966	0.0109	-0.1137	0.7930
Publishing (22)	0.0833	0.0814	-0.0214	0.0685	0.2118
Petroleum Chem. (23+24)	0.0643	0.1745	-0.0075	0.0070	0.2383
Rubber Plastics (25)	0.4358	0.0070	0.1528	0.0135	0.6091
Non-metallic prod. (26)	0.1398	0.2438	0.0147	0.0291	0.4274
Basic metals (27)	0.3501	0.4910	-0.0230	0.0176	0.8357
Metal products (28)	0.3133	0.0357	0.0139	0.0377	0.4006
Machinery (29)	0.4415	0.1713	-0.0351	0.0463	0.6240
Office machinery (30)	0.5655	-0.2873	-0.0097	0.1054	0.3738
Electrical appl. (31)	0.4152	0.0581	0.0382	-0.0042	0.5073
Radio, TV appl. (32)	0.6645	-0.0582	-0.0079	0.0606	0.6590
Medical instr. (33)	0.5117	0.0310	0.0206	-0.0141	0.5492
Motor vehicles (34)	0.2033	0.1598	-0.0219	0.0225	0.3637
Transport eq. (35)	0.3027	0.2378	0.1074	-0.0215	0.6264
Furniture (36)	0.1781	-0.0050	0.0099	0.1025	0.2855
Recycling (37)	0.4096	-0.2407	-0.1126	0.0309	0.0873
Intra industry	0.2758	0.0570	0.0126	0.0222	0.3675
Inter industry		0.0385	-0.0105	0.0057	0.0338
Total	0.2758	0.0955	0.0021	0.0279	0.4013

Notes: The aggregate TFP is obtained from a regression of log of value added on the logs of physical capital and labor using value added as weights. The decomposition is calculated for the period 1995-2000.
Source: AJPES and own calculations.

6 Conclusions

In this paper we show that two widely-used decompositions yield biased contributions of standard channels of change in the aggregate productivity. These biases can be attributed to use of ill-suited reference productivity values for entering and exiting firms in distinguishing between components measuring the contributions of surviving, entering and exiting firms, and use of fixed weights in distinguishing between contributions of productivity improvements and market share reallocation of surviving firms. In order to eliminate these biases, we propose a new decomposition that uses adequate reference productivity values in components measuring contributions of entering and exiting firms and rely on well-known Olley-Pakes decomposition. Comparison of results of different decomposition using real firm level data, we find that for fast growing sectors/industries both Foster *et al.* and Griliches and Regev decompositions attribute too large contribution to entering firms and too little contribution to surviving firms. Further, Foster *et al.* decomposition underestimates the contribution of exitors and Griliches and Regev decomposition overestimates the contribution of entrants. Moreover, lower contribution of surviving firms is reflected in lower contribution of reallocation for both of these methods. Finally, extension of our method to sectoral level suggests that intra-industry reallocation plays greater role than inter-industry reallocation, as previously suggested by Foster *et al.* (2001).

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A Appendix

Alternative decomposition with scale-independent covariance

The methods in the main text decompose the aggregate productivity index for firm-level productivity in logs. The change in aggregate productivity of variable in logs may, however, not correspond to the change in aggregate productivity of variable in levels. In case these terms differ, the aggregate productivity in levels is preferred measure as it is more closely related to the measure of welfare (Petrin and Levinsohn, 2008). The proposed decomposition in the main text is not suitable for productivity measures in levels as the market share reallocation term increases with average productivity in levels (but not in logs). Since shifts in average productivity of surviving firms must be attributed to within-firm productivity improvements, we define scale independent measure of covariance $\widetilde{cov} = cov(s, \varphi/\bar{\varphi}) = \bar{\varphi}\ cov(s, \varphi)$. Using this scale-independent covariance, we can express the relative change in the aggregate productivity as:

$$\frac{\Phi_2 - \Phi_1}{\bar{\Phi}} = (1 + \overline{\widetilde{cov}}_S)\frac{\Delta\bar{\varphi}_S}{\bar{\Phi}} + \frac{\bar{\bar{\varphi}}_S}{\bar{\Phi}}\Delta\widetilde{cov}_S + s_{E2}\frac{\Phi_{S2} - \Phi_{E2}}{\bar{\Phi}} + s_{X1}\frac{\Phi_{S1} - \Phi_{X1}}{\bar{\Phi}}, \quad (17)$$

where $\bar{\Phi} = 1/2(\Phi_1 + \Phi_2)$, $\overline{\widetilde{cov}}_S = 1/2(\widetilde{cov}_{S2} + \widetilde{cov}_{S1})$ and $\bar{\bar{\varphi}}_S = 1/2(\bar{\varphi}_{S1} + \bar{\varphi}_{S2})$. Unlike the decomposition for productivity in logs, this decomposition weights the contributions of average productivity improvement of surviving firms with average scale-independent covariance and change in covariance by the time-average of unweighted productivity of surviving firms.

Table 8: Dynamic Olley-Pakes decomposition with entry and exit

Log of valued added per employee							
	Surviving firms		Entering firms		Exiting firms		
Year	Change in Unweighted Productivity	Change in Covariance	Unweighted Productivity Difference Ent. vs. Sur.	Weighted Productivity Difference Ent. vs. Sur.	Unweighted Productivity Difference Sur. vs. Exit.	Weighted Productivity Difference Sur. vs. Exit.	Total
1996	0.0657	0.0670	-0.0006	-0.0166	0.0434	0.0428	0.1356
1997	0.1727	0.1167	0.0056	-0.0230	0.0758	0.0759	0.3013
1998	0.2386	0.0697	-0.0192	-0.0258	0.0931	0.0990	0.3307
1999	0.2888	0.1162	-0.0103	-0.0241	0.0887	0.0898	0.4223
2000	0.2996	0.1623	-0.0078	-0.0195	0.0972	0.0971	0.4865

Notes: The aggregate value added per employee is calculated with employment shares as weights.
The reference period for calculation of the change of aggregate productivity index is 1995.
Source: AJPES and own calculations.

In order to evaluate the differences between decompositions of weighted average productivity in levels and in logs, we also estimate the components of decomposition for value added per employee with employment shares as weights. The results of this decomposition are shown in Table 8 and are directly comparable the results in the top panel of Table 6. Note that the relative change in aggregate value per employee, $(\Phi_2 - \Phi_1)/\bar{\Phi}$, is slightly lower than the change in weighted log of value per employee, although the difference is increasing with the length of time span. The contributions of four channels are also fairly similar, with greater contributions of market share reallocation between survivors and exitors and lower contribution of productivity improvements of surviving firms. This difference is mainly the result of negative covariance between market share and scale independent value added per employee, which is reflected in the first and fourth terms of decomposition (17). While the two decompositions give fairly similar results, the decomposition for productivity in levels may be preferred for longer time span.